Supporting Information for "Undulated Shock Surface Formed After a Shock–Discontinuity Interaction"

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Introduction This supporting information gives a brief description of the method of normal field analysis (NFA) which aims at estimating the principal directions and curvatures of a surface in space based on at least four-point measurement.

Text S1.

For a surface s and its normal field $\mathbf{n}_s(\mathbf{x})$, the principal directions of the surface at a given point $\mathbf{x}_0$, either by definition or in terms of Rodrigues' Theorem, are the eigenvectors of the Weingarten map (or shape operator) $-\nabla_{\mathrm{t}}\mathbf{n}_s(\mathbf{x})|_{\mathbf{x}_0}$, where ∇_{t} is the gradient operator in the tangent plane at $\mathbf{x}_0$. The principal curvatures are the associated eigenvalues. Therefore, the gradient of normal field, i.e. the negated Weingarted map, can be cast into matrix form with principal directions as the set of basis:

$$\nabla_{\mathrm{t}}\mathbf{n}_s = \begin{pmatrix} -\kappa_1 & 0 \\ 0 & -\kappa_2 \end{pmatrix} . \tag{1}$$

Note that we follow the commonly defined principal curvatures in differential geometry so that minus signs appear before the curvatures to make a minor difference with the original NFA method.

The surfaces in space may generally move at a relative speed to the constellation of spacecraft and crossed each spacecraft at a different time. If the surface is stable during the crossing event, i.e. not deformed too heavily from its first encounter of a spacecraft to its final encounter of the last spacecraft, the motion of the surface fills the local space up with a series of virtual surfaces, each corresponding to a snapshot of the surface at a time (Fig. S1). Inversely, for each point $\mathbf{x}$ in the local space there must uniquely exist one surface containing the point. With each of these surfaces is affiliated a field of normal as the collection of normals at all surface elements. In other words, given a surface s defined by

$$F_s\left(\mathbf{x}\right) = 0, \tag{2}$$

we have a normal field $\mathbf{n}_s(\mathbf{x})$ where $\mathbf{x}$ satisfy the surface equation (2). Such normal field is on a two-dimensional surface. The amalgamation of normal fields $\mathbf{n}_s(\mathbf{x})$ on the series of surfaces leads to an aggregated normal field $\mathbf{n}(\mathbf{x})$ in three-dimensional space, which is defined by

$$\mathbf{n}(\mathbf{x}) = \mathbf{n}_s(\mathbf{x})\,. \tag{3}$$

Supposing $\mathbf{x}$ is on surface s, then the aggregated normal field at this point equal the normal of surface s at it. This definition naturally leads to

$$\nabla_{\mathrm{t}}\mathbf{n} = \nabla_{\mathrm{t}}\mathbf{n}_s\,. \tag{4}$$

Therefore, to estimate the principal directions and curvatures, we need the two dimensional gradient of the three dimensional normal fields $\nabla_{\mathrm{t}}\mathbf{n}$ which is obtained by eliminating the derivative along $\mathbf{n}$:

$$\nabla_{\mathrm{t}}\mathbf{n} = \nabla\mathbf{n} - \mathbf{n}\left(\mathbf{n}\cdot\nabla\mathbf{n}\right), \tag{5}$$

Since normal field is a unit vector field, any derivative of it must be perpendicular to it, which means

$$\nabla\mathbf{n}\cdot\mathbf{n} = (0,0,0)\,.$$

Therefore, $\nabla_{\mathrm{t}}\mathbf{n}$ would have two non-vanishing eigenvalues $-\kappa_1$ and $-\kappa_2$, whose corresponding eigenvectors are the principal directions. The last eigenvalue is zero, and the eigenvector is the local normal. In its eigenbasis, $\nabla_{\mathrm{t}}\mathbf{n}$ takes the form:

$$\nabla_{\mathrm{t}}\mathbf{n} = \begin{pmatrix} -\kappa_1 & 0 & 0 \\ 0 & -\kappa_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{6}$$

By solving for its eigensystem, we obtain the principal directions and curvatures of the local surface.

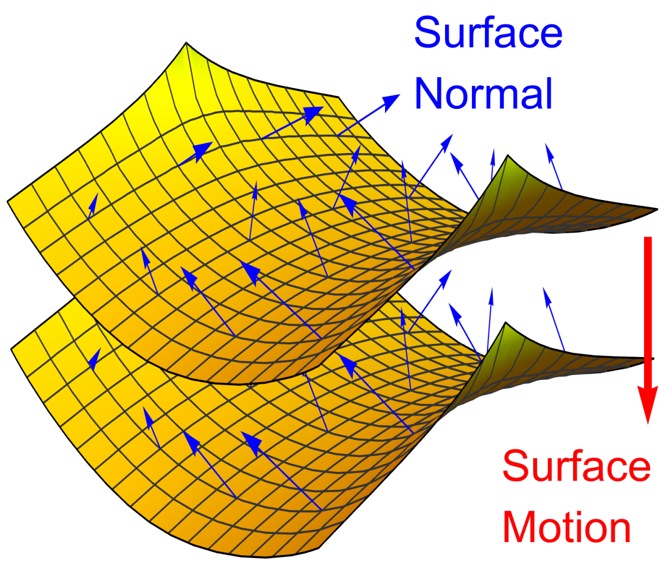

Figure S1. Illustration on the normal field produced by the motion of a surface.